Final Exam College Algebra
Fall Semester 2015
Version A

Instructor: Name:
Section: Student Verification:

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Time Limit: 120 minutes
No graphing calculators may be used. No notes, books, cell phones, or any devices that can connect to the Internet are allowed.
Scientific calculators with no more than a basic numeric store and recall memory are allowed on the final exam.
Reference formulas that are allowed are attached to the exam.
This exam has two parts:
Part I - Five multiple-choice questions worth up to 4 points each.
Part II – Seven open ended questions. Be sure to answer each part. Points possible for each question will vary.

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Part I: Multiple-Choice. Partial credit may be awarded depending on the answer chosen.

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1. Solve the equation:

$\log x + \log(x+3) = 1.$

A. 5

B. 5, −2

C. −5, 2

D. 2

E. $\frac{3+\sqrt{13}}{2}$

F. No solution

Domain $x > 0$ & $x+3 > 0$, so $x > -3$, which means $x > 0$.

$\log x + \log(x+3) = 1$

$\log[x(x+3)] = 1$

$10^1 = x(x+3)$

$10 = x^2 + 3x$

$0 = x^2 + 3x - 10$

$0 = (x+5)(x-2)$

$x = -5$ (not in domain) $x = 2$ (solution) Check it!

fix this

2. Solve the system of linear equations given in augmented matrix form:

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & -1 & 1 \\ 2 & -1 & 4 & 2 \end{array}\right]$

A. {(6, 3, −2)}

B. {(6, −3, −2)}

C. {(6, −2, −3)}

D. {(1, −1, 3)}

E. {(2, 2, −3)}

F. {(−2, 1, 3)}

Use row operations

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & -1 & 1 \\ 2 & -1 & 4 & 2 \end{array}\right]$ $-2R_1 + R_2 \to R_2$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & -3 & 4 & -6 \end{array}\right]$ $-R_2 + R_1 \to R_1$, $3R_2 + R_3 \to R_3$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -3 \end{array}\right]$ $-R_3 + R_1 \to R_1$, $R_3 + R_2 \to R_2$ $\to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \end{array}\right]$

(6, −2, −3)

3. Find the sum of the geometric series: $\frac{3}{2} - \frac{3}{4} + \frac{3}{8} - \frac{3}{16} + \cdots$.

A. 1

B. −1

C. 3

D. $\frac{15}{16}$

E. $\frac{3}{2}$

F. ∞

and form

Infinite geom. series: $S = \frac{a}{1-r}$

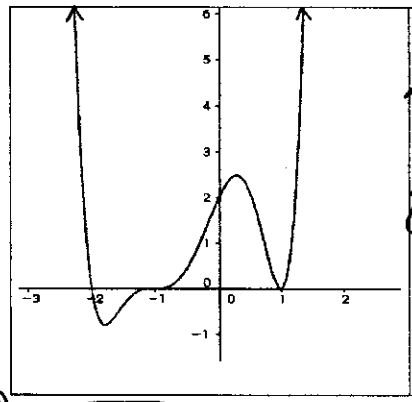
$a = \frac{3}{2}$

$r = -\frac{1}{2}$

$S = \frac{\frac{3}{2}}{1-(-\frac{1}{2})} = \frac{3/2}{3/2} = 1$

6 → Any form: power function, zeros mult., touch, cross, y-int, graph.

4. Below is the graph of a polynomial function $P(x)$. Looking at the properties of the graph (i.e. end behavior, zeros, multiplicity, and intercepts). Which equation below best describes $P(x)$?



$P(0) = 2(1)(1)(2) = 4$

~~A.~~ $P(x) = 2(x-1)^2(x+1)^3(x+2)$

~~B.~~ $P(x) = (x+1)^2(x+2)(x-1)^2$

~~C.~~ $P(x) = -2(x-1)^2(x+1)^3(x+2)$

~~D.~~ $P(x) = (x+1)^2(2-x)(x+2)(1-x)$

~~E.~~ $P(x) = (x-1)^2(x+1)(x-2)$

F. $P(x) = (x-1)^2(x+1)^3(x+2)$

$P(0) = (1)(1)(2) = 2$ ✓

Both ends up → even degree & positive leading coefficient

Zero at $x=-2$ → odd mult.

$x=-1$ → odd mult.

$x=1$ → even mult.

y-int: $(0, 2)$

factors: $(x+2)^{odd}(x+1)^{odd}(x-1)^{even}$

$p(0) = 2$

5. Write the partial fraction decomposition of the rational expression.

$$\frac{6x-48}{(x-4)(x+2)}$$

A. $\frac{6x}{x+2} - \frac{48}{x-4}$

B. $\frac{4}{x+2} - \frac{10}{x-4}$

C. $\frac{6x}{x+2} + \frac{48}{x-4}$

D. $\frac{10}{x+2} - \frac{4}{x-4}$

E. $\frac{7}{x-4} - \frac{2}{x+2}$

F. $\frac{42x}{x^2-2x-8}$

$$\frac{6x-48}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$$

$6x-48 = A(x+2) + B(x-4)$

let $x=-2$: $6(-2)-48 = B(-2-4)$

$-12-48 = -6B$

$-60 = -6B$

$10 = B$

$\frac{10}{x+2}$

let $x=4$: $6(4)-48 = A(4+2)$

$24-48 = 6A$

$-24 = 6A$

$-4 = A$

$\frac{-4}{x-4}$

Part II: Be sure you answer each part of the problem. Partial credit may be awarded. Show appropriate work.

6. Given the function, answer the following. $f(x) = 2x^2 - 4x - 3$

Find the vertex. (Leave your answer as an ordered pair.) Box your final answer.

Vertex: (h, k)

$h = -\frac{b}{2a}$ $\quad h = \frac{4}{2 \cdot 2} = \frac{4}{4} = 1$

$k = f(h) = f(1) = 2(1)^2 - 4(1) - 3$

$= 2 - 4 - 3$

$= -5$

$\boxed{(1, -5)}$

Find the exact x-intercept(s).

$0 = 2x^2 - 4x - 3$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$

$x = \frac{4 \pm \sqrt{16+24}}{4} = \frac{4 \pm \sqrt{40}}{4}$

$x = \frac{4 \pm 2\sqrt{10}}{4}$

Find the y-intercept.

$f(0) = 2(0)^2 - 4(0) - 3$

$= -3$

$\boxed{(0, -3)}$

Find the domain of $f(x)$.

Domain: $(-\infty, \infty)$

Find the range of $f(x)$.

Range:

$[-5, \infty)$

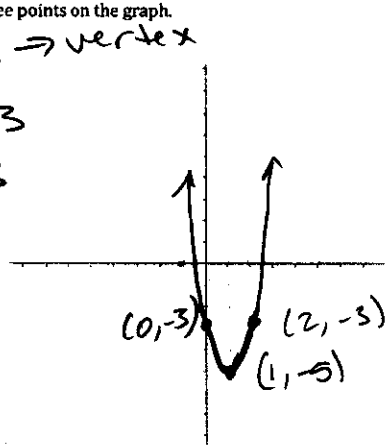
6. Continued: Given the function, answer the following. $f(x) = 2x^2 - 4x - 3$

Graph $f(x)$. Label three points on the graph.

$f(1) = -5 \Rightarrow$ vertex

$f(0) = -3$

$f(2) = -3$



Find the difference quotient of $f(x) = 2x^2 - 4x - 3$ where $\frac{f(x+h) - f(x)}{h}$. Simplify and box your answer.

$f(x+h) = 2(x+h)^2 - 4(x+h) - 3$

$= 2(x^2 + 2xh + h^2) - 4x - 4h - 3$

$= 2x^2 + 4xh + 2h^2 - 4x - 4h - 3$

$-f(x) = -2x^2 + 4x + 3$

$\frac{f(x+h)-f(x)}{h} = \frac{2x^2 + 4xh + 2h^2 - 4x - 4h - 3 - 2x^2 + 4x + 3}{h}$

$\frac{4xh + 2h^2 - 4h}{h} = 4x + 2h - 4$

7. Given the function below, answer the following:

$R(x) = \frac{-4x}{x^2 - 36}$

Find the x-intercept(s).

$0 = \frac{-4x}{x^2 - 36}$

$-4x = 0$

$x = 0$

$(0,0)$

Find the y-intercept.

$R(0) = \frac{-4(0)}{(0)^2 - 36} = \frac{0}{-36} = 0$

$(0,0)$

Find the domain of $R(x)$.

$x^2 - 36 \neq 0$

$(x-6)(x+6) \neq 0$

$x \neq 6 \quad x \neq -6$

$(-\infty, -6) \cup (-6, 6) \cup (6, \infty)$

Find any vertical asymptotes. (If not applicable, state so). Leave your answers as an equation.

$x = 6$

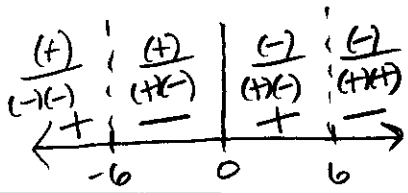
$x = -6$

Find any horizontal asymptotes. (If not applicable, state so). Leave your answers as an equation.

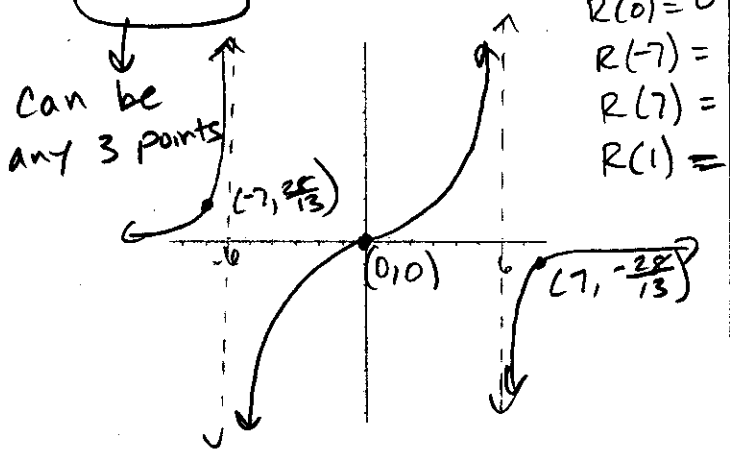
$y = 0$

7 Continued: Given the function below, answer the following:

$$R(x) = \frac{-4x}{x^2 - 36} \qquad (x+6)(x-6)$$



Graph $R(x)$. Label three points on the graph. Show all asymptotes.



$R(0) = 0$

$R(-7) = \frac{28}{13}$

$R(7) = -\frac{28}{13}$

$R(1) = \frac{-4}{-35} = \frac{4}{35}$

$R(-1) = -\frac{4}{35}$

Is $R(x)$ even, odd, or neither? Support this answer with sound algebraic reasoning.

Even if $R(x) = R(-x)$

odd if $R(-x) = -R(x)$

$$R(-x) = \frac{-4(-x)}{(-x)^2 - 36} = \frac{4x}{x^2 - 36}$$

$$-R(x) = \frac{4x}{x^2 - 36}$$ ← same sp

odd

8 Given the function below, answer the following: $f(x) = 2^{x-1}$

Find f^{-1}

$y = 2^{x-1}$

$x = 2^{y-1}$

$\log_2 x = y - 1$

$\log_2 x + 1 = f^{-1}(x)$

Find the domain of f $(-\infty, \infty)$	Find the range of f $(0, \infty)$
Find the domain of f^{-1} $(0, \infty)$	Find the range of f^{-1} $(-\infty, \infty)$
Find the y-intercept of f $f(0) = 2^{0-1} = 2^{-1} = 1/2$ $(0, 1/2)$	Find the x-intercept of f^{-1} $(1/2, 0)$
Find the horizontal asymptote of f $y = 0$	Find the vertical asymptote of f^{-1} $x = 0$

8. Continued: Given the function below, answer the following: $f(x) = 2^{x-1}$

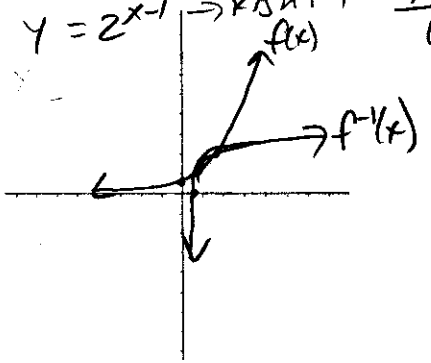
Graph f and f^{-1} on the same coordinate plane. Label both functions.

Parent function for $f(x)$.

$y = 2^x$

x	y
-1	1/2
0	1
1	2

$y = 2^{x-1}$ → Right 1

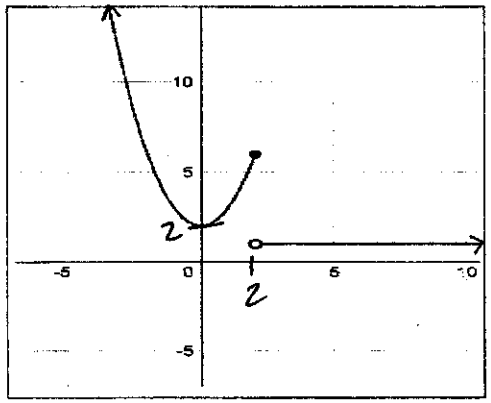


f(x)	
x+1	y
0	1/2
1	1
2	2

$f^{-1}(x)$

x	$f^{-1}(x)$
1/2	0
1	1
2	2

9. Given the graph of a piece-wise function $G(x) = \begin{cases} x^2 + 2, & x \le 2 \\ 1, & x > 2 \end{cases}$

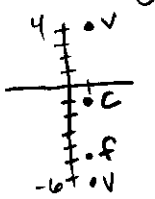


Find the domain of $G(x)$. $(-\infty, \infty)$	Find the range of $G(x)$. $\{1\} \cup [2, \infty)$ or $[1] \cup [2, \infty)$
Find $G(2)$ $G(2) = (2)^2 + 2 = 6$	Determine the interval where the function is increasing. $(0, 2)$
Find $G(-3)$ $G(-3) = (-3)^2 + 2 = 11$	Determine the interval where the function is decreasing. $(-\infty, 0)$
Find $G\left(\frac{3}{2}\right)$ $G\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 + 2 = \frac{9}{4} + 2 = \frac{17}{4}$	Determine the interval where the function is constant. $(2, \infty)$

Parts

$\frac{4+-6}{2} = \frac{-2}{2} = -1$

10. Find the equation of an ellipse where the vertices are at (1, 4) and (1, −6) and a focus is at (1, −5).



$c(1,-1)$ major axis is vertical

$a = 5$

$c = 4$

$b = 3$

$a^2 = b^2 + c^2$

$25 = b^2 + 16$

$9 = b^2 \quad b = 3$

$$\frac{(x-1)^2}{25} + \frac{(y+1)^2}{9} = 1$$

11. The half-life of Actinium (Ac-227) is 21.77 years. How long does it take 40 grams of Actinium to decay to 26 grams? Approximate your final answer to two decimals.

$A = A_0 e^{kt}$ find k first

(1) $\frac{1}{2} = e^{k(21.77)}$

$\frac{\ln(\frac{1}{2})}{21.77} = \frac{21.77k}{21.77}$

$k = \frac{\ln(.5)}{21.77}$

↓ use this exact value to find t.

(2) $26 = 40 e^{\frac{\ln(.5)}{21.77} t}$

$\frac{26}{40} = e^{\frac{\ln(.5)}{21.77} t}$

$\ln(\frac{13}{20}) = \frac{\ln(.5)}{21.77} t$

$\frac{\ln(\frac{13}{20})}{(\frac{\ln(.5)}{21.77})} = t$

$13.53 \text{ years} \approx t$

12. Given $f(x) = x + 2$ and $g(x) = \frac{x-9}{x}$ $x \neq 0$

(fog)(x) or (gof)(x)

a) Find the composite function $(f \circ g)(x)$

$f(g(x)) = f(\frac{x-9}{x})$

$= \frac{x-9}{x} + 2$

$= \frac{x-9}{x} + \frac{2x}{x}$

$= \frac{3x-9}{x}$

b) What is the domain of $(f \circ g)(x)$

Domain: $x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

c) Evaluate $(f \circ g)(3)$

$f(g(3)) = \frac{3(3)-9}{3} = \frac{0}{3} = 0$